

DIFFERENTIAL EQUATIONS

EXERCISE 2.10

Problems solved by;

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~~VARIABLES~~ the solution of the equation.

$$y'' + py' + qy = r(x) \quad \text{--- (1)}$$

where p , q and r are functions of x which are continuous on some interval I . The method gives a particular solution of (1) on I in the form.

$$y_p(x) = -y_1 \int \frac{y_2 r(x) dx}{W} + y_2 \int \frac{y_1 r(x) dx}{W} \quad \text{--- (2)}$$

where y_1 and y_2 form a basis of the solutions of homogeneous equations.

$y'' + py' + qy = 0$ --- (3) corresponding to (1) and W is the Wronskian of y_1 and y_2 .

PROOF

Suppose that linearly independent solutions of (3) are $y = y_1$ and $y = y_2$. Then.

$$y_h = c_1 y_1 + c_2 y_2$$

with c_1 and c_2 arbitrary constants. We replace these arbitrary constants by unknown functions $u_1(x)$ & $u_2(x)$, and require that

$$y_p = u_1 y_1 + u_2 y_2 \quad \text{--- (4)}$$

be the particular solution of (1)

In order to determine u_1 and u_2 we need two conditions. One condition is that (4) must satisfy (1). A second condition we can impose arbitrarily.

Differentiating (4) w.r.t x we get.

$$y_p' = u_1' y_1 + u_2' y_2 + u_1 y_1' + u_2 y_2'$$

If we further differentiate this y_p' contains u_1'' and u_2'' . To avoid 2nd derivatives of u_1 and u_2 , we set.

$$u_1' y_1 + u_2' y_2 = 0 \quad \text{--- (5)}$$

$$y_p' = u_1 y_1' + u_2 y_2'$$

$$\Rightarrow y_p'' = u_1' y_1' + u_1 y_1'' + u_2' y_2' + u_2 y_2''$$

Substituting for y_p , y_p' and y_p'' into (1)

$$\Rightarrow (u_1' y_1' + u_1 y_1'' + u_2' y_2' + u_2 y_2'') + P(u_1 y_1' + u_2 y_2') + Q(u_1 y_1 + u_2 y_2) = r(x)$$

$$\Rightarrow u_1 (y_1'' + P y_1' + Q y_1) + u_2 (y_2'' + P y_2' + Q y_2) + u_1' y_1' + u_2' y_2' = 0$$

$$y_1'' + P y_1' + Q y_1 = 0 \text{ because } y_1 \text{ is a sol. of (3)}$$

$$\text{Similarly } y_2'' + P y_2' + Q y_2 = 0$$

$$\Rightarrow u_1' y_1' + u_2' y_2' = r(x) \text{ --- (6)}$$

Taking (4) and (5) together we have

$$u_1 y_1' + u_2 y_2' = 0$$

$$u_1 y_1' + u_2 y_2' = r(x)$$

Solving these, we have

$$\left. \begin{aligned} u_1' &= \frac{-y_2 r(x)}{y_1 y_2' - y_1' y_2} = \frac{-y_2 r(x)}{W} \\ u_2' &= \frac{y_1 r(x)}{y_1 y_2' - y_1' y_2} = \frac{y_1 r(x)}{W} \end{aligned} \right\} \text{--- (7)}$$

In (7) $W \neq 0$ since y_1 and y_2 are linearly independent solutions of (3)
by Integrating we get.

$$u_1 = \int \frac{-y_2 r(x)}{W} dx \quad \& \quad u_2 = \int \frac{y_1 r(x)}{W} dx$$

$$\Rightarrow y_p = -y_1 \left(\int \frac{y_2 r(x)}{W} dx \right) + y_2 \left(\int \frac{y_1 r(x)}{W} dx \right)$$

parameters because here we have change the parameter (i.e. c_1 and c_2 by u_1 and u_2).

EXERCISE 2.10

Find General Solution of Non-homogeneous Eqs

$$\textcircled{1} \quad y'' - 4y' + 4y = \frac{e^{2x}}{x}. \quad \text{---} \textcircled{1}$$

Sol For y_h , we have the characteristic eq of the corresponding homogeneous linear eq as

$$\lambda^2 - 4\lambda + 4 = 0 \Rightarrow \lambda^2 - 2\lambda - 2\lambda + 4 = 0 \Rightarrow \lambda(\lambda - 2) - 2(\lambda - 2) = 0$$

$$\Rightarrow \lambda = 2, 2. \quad (\text{double roots})$$

Hence.

$$y_h = (c_1 + c_2 x) e^{2x}.$$

$$\text{So } y_1 = e^{2x} \text{ \& } y_2 = x e^{2x}.$$

$$W = \begin{vmatrix} e^{2x} & x e^{2x} \\ 2e^{2x} & e^{2x} + 2x e^{2x} \end{vmatrix} = \frac{e^{4x} - 2x e^{4x} - 2x e^{4x}}{e^{4x}} = e^{4x}.$$

So Particular Sol of $\textcircled{1}$ is.

$$\Rightarrow y_p = -y_1 \int \frac{y_2 r(x)}{W} dx + y_2 \int \frac{y_1 r(x)}{W} dx.$$

$$\Rightarrow y_p = -e^{2x} \int \frac{x e^{2x} (e^{2x}/x)}{e^{4x}} dx + x e^{2x} \int \frac{e^{2x} (e^{2x}/x)}{e^{4x}} dx$$

$$\Rightarrow y_p = -x e^{2x} + x e^{2x} \ln x.$$

$$\therefore y = y_h + y_p.$$

$$= (c_1 + c_2 x - x + x \ln x) e^{2x} \quad \underline{\text{Ans.}}$$

sol For y_h , we have.

$$\lambda^2 + 9 = 0 \Rightarrow \lambda = \pm 3i. \quad (\text{CASE - III})$$

$$\text{So } y_h = A \cos 3x + B \sin 3x.$$

Q2

$$W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3 \sin 3x & 3 \cos 3x \end{vmatrix} = 3 \cos^2 3x + 3 \sin^2 3x = 3.$$

$$\therefore y_p = -\cos 3x \int \frac{\sin 3x \cdot \sec 3x}{3} dx + \sin 3x \int \frac{\cos 3x \cdot \sec 3x}{3} dx$$

$$\Rightarrow y_p = \frac{1}{9} \cos 3x \ln \cos 3x + \frac{1}{3} \sin 3x. \quad \underline{\text{Ans}}$$

$$\textcircled{3} y'' + 2y' + y = e^{-x} \cos x$$

sol For y_h , we have

$$\lambda^2 + 2\lambda + 1 = 0.$$

$$\Rightarrow \lambda^2 + \lambda + \lambda + 1 = 0 \Rightarrow \lambda(\lambda + 1) + 1(\lambda + 1) = 0$$

$$\Rightarrow \lambda = -1, -1. \quad (\text{CASE II})$$

$$\text{hence } y_h = (c_1 + c_2 x) e^{-x}.$$

$$W = \begin{vmatrix} e^{-x} & x e^{-x} \\ -e^{-x} & e^{-x} - x e^{-x} \end{vmatrix} = e^{-2x} - x e^{-2x} + x e^{-2x} = e^{-2x}.$$

$$\therefore y_p = -e^{-x} \int \frac{x e^{-x} \cdot e^x \cos x}{e^{-2x}} dx + x e^{-x} \int \frac{e^{-x} e^x \cos x}{e^{-2x}} dx$$

$$\Rightarrow y_p = -e^{-x} \left[x \sin x - \int \sin x dx \right] + x e^{-x} \sin x.$$

$$\Rightarrow y_p = -x e^{-x} \sin x - e^{-x} \cos x + x e^{-x} \sin x$$

$$\Rightarrow y_p = -e^{-x} \cos x.$$

$$\therefore y = y_h + y_p = (c_1 + c_2 x + \cos x) e^{-x}.$$

Ans

For y_h , we have $\lambda^2 + 9 = 0 \Rightarrow \lambda = \pm 3i$.

$$\Rightarrow y_h = A \cos 3x + B \sin 3x.$$

Now

$$Q4 \quad W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = 3.$$

$$\Rightarrow y_p = -\cos 3x \int \frac{\sin 3x \cdot \csc 3x}{3} dx + \sin 3x \int \frac{\cos 3x \cdot \csc 3x}{3} dx$$

$$\Rightarrow y_p = -\frac{x \cos 3x}{3} + \frac{1}{9} \sin 3x \ln |\sin 3x|.$$

Hence

$$y = A \cos 3x + B \sin 3x - \frac{x}{3} \cos 3x + \frac{1}{9} \sin 3x \ln |\sin 3x| \quad \text{Ans.}$$

$$(5) \quad y'' - 2y' + y = x^2/x^3.$$

For y_h we have

$$\lambda^2 - 2\lambda + 1 = 0 \Rightarrow \lambda^2 - \lambda - \lambda + 1 = 0 \Rightarrow \lambda(\lambda - 1) - 1(\lambda - 1) = 0$$

$$\Rightarrow \lambda = 1, 1 \quad (\text{CASE II})$$

$$\Rightarrow y_h = (c_1 + c_2 x) e^x.$$

$$W = \begin{vmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{vmatrix} = e^{2x} + xe^{2x} - xe^{2x} = e^{2x}.$$

$$\Rightarrow y_p = -e^x \int \frac{x^2/x^3}{e^{2x}} dx + xe^x \int \frac{e^x e^x/x^3}{e^{2x}} dx$$

$$\Rightarrow y_p = -e^x \frac{1}{-x} + xe^x \frac{1}{-2x^2} = \frac{e^x}{x} - \frac{e^x}{2x}.$$

$$\Rightarrow y = y_h + y_p = (c_1 + c_2 x + \frac{1}{x} - \frac{1}{2x}) e^x.$$

$$\Rightarrow y = (c_1 + c_2 x + \frac{1}{2} x) e^x \quad \text{Ans}$$

$$(6) \quad y'' - 4y' + 5y = e^{2x} \csc x$$

For y_h , we have.

$$\lambda^2 - 4\lambda + 5 = 0 \Rightarrow \lambda = \frac{4 \pm \sqrt{16 - 20}}{2} = \frac{4 \pm 2i}{2}$$

$$\Rightarrow \lambda = 2 \pm i \quad (\text{CASE III})$$

$$\Rightarrow y_h = e^{2x} (A \cos x + B \sin x).$$

Now

$$W = \begin{vmatrix} e^{2x} \cos x & e^{2x} \sin x \\ 2e^{2x} \cos x - e^{2x} \sin x & 2e^{2x} \sin x + e^{2x} \cos x \end{vmatrix}$$

$$\begin{aligned} \Rightarrow W &= 2e^{4x} \sin x \cos x + e^{4x} \cos^2 x - 2e^{4x} \sin x \cos x \\ &\quad + e^{4x} \sin^2 x \\ \Rightarrow W &= e^{4x} \end{aligned}$$

$$\therefore y_p = -e^{2x} \cos x \int \frac{e^{1/2} \sin x \cdot e^{1/2} \cos x \cdot e^{1/2} dx}{e^{4x}} + e^{2x} \sin x \int \frac{e^{1/2} \cos x \cdot e^{1/2} \sin x \cdot e^{1/2} dx}{e^{4x}}$$

$$\Rightarrow y_p = -xe^{2x} \cos x + e^{2x} \sin x \ln \sin x.$$

$$\therefore y = y_h + y_p = (A \cos x + B \sin x - x \cos x + \sin x \ln \sin x) e^{2x}$$

Ans

⑦ $(D^2 - 2D + 1)y = 3x^{3/2}e^x$

For y_h , we have.

$$\lambda^2 - 2\lambda + 1 = 0 \Rightarrow \lambda^2 - \lambda - \lambda + 1 = 0 \Rightarrow \lambda(\lambda - 1) - 1(\lambda - 1) = 0$$

$$\Rightarrow \lambda = 1, 1 \quad (\text{CASE II})$$

$$\therefore y_h = (C_1 + C_2 x) e^x$$

$$W = \begin{vmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{vmatrix} = e^{2x} + xe^{2x} - xe^{2x} = e^{2x}$$

$$\therefore y_p = -e^x \int \frac{x^{3/2} 3x^{3/2} e^x}{e^{2x}} dx + xe^x \int \frac{x^{3/2} 3x^{3/2} e^x}{e^{2x}} dx$$

$$\Rightarrow y_p = \frac{6e^x \cdot x^{7/2}}{7} + \frac{6e^x x^{7/2}}{5}$$

$$\therefore y = y_h + y_p = (C_1 + C_2 x + \frac{6}{7} + \frac{6}{5} x) e^x$$

$$\Rightarrow y = (C_1 + C_2 x + \frac{12}{35} x^{7/2}) e^x$$

Ans

⑧ $(D^2 + 6D + 9)y = 16e^{-3x} / (x^2 + 1)$

For y_h , we have.

$$\lambda^2 + 6\lambda + 9 = 0 \Rightarrow \lambda^2 + 3\lambda + 3\lambda + 9 = 0$$

$$\Rightarrow \lambda(\lambda + 3) + 3(\lambda + 3) \Rightarrow \lambda = -3, -3 \quad (\text{CASE II})$$

$$\therefore y_h = (C_1 + C_2 x) e^{-3x}$$

$$\begin{aligned}
 & \begin{vmatrix} -3e^{-3x} & e^{-3x} - 3xe^{-3x} \end{vmatrix} = e^{-3x} - 3xe^{-3x} + 3xe^{-3x} = e^{-6x} \\
 & y_h = -e^{-3x} \int \frac{xe^{-3x} \cdot 6e^{-3x} (x^2+1)}{e^{-6x}} dx + xe^{-3x} \int \frac{e^{-3x} \cdot e^{-3x}}{e^{-6x}} dx \\
 & \Rightarrow y_p = -\frac{1}{2} e^{-3x} \int \frac{2x}{x^2+1} dx + xe^{-3x} \int \frac{dx}{x^2+1} \\
 & \Rightarrow y_p = -\frac{1}{2} e^{-3x} \ln|x^2+1| + xe^{-3x} \tan^{-1} x \\
 & \therefore y = y_h + y_p = (C_1 + C_2 x - \frac{1}{2} \ln|x^2+1| + x \tan^{-1} x) e^{-3x} \\
 & \quad \quad \quad \text{Ans} \\
 & \textcircled{9} (D^2 + 4D + 4)y = 2e^{-2x}/x^2 \\
 & \text{For } y_h, \text{ we have} \\
 & \lambda^2 + 4\lambda + 4 = 0 \Rightarrow \lambda + 2\lambda + 2\lambda + 4 = 0 \\
 & \Rightarrow \lambda(\lambda+2) + 2(\lambda+2) \Rightarrow \lambda = -2, -2 \text{ (CASE II)} \\
 & \text{hence } y_h = (C_1 + C_2 x) e^{-2x} \\
 & \text{Also } W = \begin{vmatrix} e^{-2x} & xe^{-2x} \\ -2e^{-2x} & e^{-2x} - 2xe^{-2x} \end{vmatrix} = e^{-4x} - 2xe^{-4x} + 2xe^{-4x} = e^{-4x} \\
 & \therefore y_p = -e^{-2x} \int \frac{xe^{-2x} \cdot 2e^{-2x}/x^2}{e^{-4x}} dx + xe^{-2x} \int \frac{e^{-2x} \cdot 2e^{-2x}/x^2}{e^{-4x}} dx \\
 & \Rightarrow y_p = -2e^{-2x} \ln|x| + 2xe^{-2x} \left(-\frac{1}{x}\right) \\
 & \Rightarrow y_p = -2e^{-2x} \ln|x| - 2e^{-2x} \\
 & \therefore y = y_h + y_p = (C_1 + C_2 x) e^{-2x} - (2 \ln|x| + 2) e^{-2x} \\
 & \quad \quad \quad \text{Ans} \\
 & \textcircled{10} (D^2 + 2D + 2)y = 4e^{-x} \sec^3 x \\
 & \text{For } y_h, \text{ we have} \\
 & \lambda^2 + 2\lambda + 2 = 0 \Rightarrow \lambda + \lambda + 2 = 0 \\
 & \Rightarrow \lambda = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i \text{ (CASE - III)} \\
 & \therefore y_h = e^{-x} (A \cos x + B \sin x)
 \end{aligned}$$

$$\begin{aligned}
 & \left[-e^x \cos x - e^x \sin x - e^x \sin x + e^x \cos x \right] \\
 \Rightarrow W &= -e^{-2x} \sin x \cos x + e^{-2x} \cos^2 x + e^{-2x} \sin x \cos x + e^{-2x} \sin^2 x \\
 \Rightarrow W &= e^{-2x} \\
 \Rightarrow y_p &= -e^x \cos x \int \frac{e^x \sin x \cdot \frac{1}{2} e^x \sec^2 x}{e^{2x}} dx + e^x \sin x \int \frac{e^x \cos x \sec^2 x}{e^{2x}} dx \\
 \Rightarrow y_p &= -e^x \cos x \int \sec x \cdot \sec x \tan x dx + e^x \sin x \int \sec x dx \\
 \Rightarrow y_p &= -e^x \cos x \int \sec x \frac{d}{dx} \sec x dx + e^x \sin x \tan x \\
 \Rightarrow y_p &= -e^x \cos x \frac{\sec^2 x}{2} + e^x \sin x \tan x \\
 \Rightarrow y_p &= -\frac{1}{2} e^x \sec x + e^x \sin x \tan x \\
 \text{or } y_p &= -e^x \cos x \int \sec^2 x \tan x dx + e^x \sin x \int \sec^2 x dx \\
 \Rightarrow y_p &= -e^x \cos x \frac{\tan^2 x}{2} + e^x \sin x \tan x \\
 \Rightarrow y_p &= -\frac{1}{2} e^x \sin^2 x \sec x + e^x \sin^2 x \sec x \\
 \Rightarrow y_p &= \frac{1}{2} e^x \sin^2 x \sec x \\
 \therefore y &= \left(A \cos x + B \sin x + \frac{1}{2} \sin^2 x \sec x \right) e^{-x} \quad \underline{\underline{\text{Ans}}}
 \end{aligned}$$

Find G.S of the following Non-homogeneous Euler Cauchy equations.

⑪ $x^2 y'' - 4xy' + 6y = 21x^{-4}$ ——— ①

for y_h , we have

$$m^2 + (a-1)m + b = 0.$$

$$\Rightarrow m^2 + (-5)m + 6 = 0$$

$$\Rightarrow m^2 - 5m + 6 = 0$$

$$\Rightarrow m^2 - 3m - 2m + 6 = 0 \Rightarrow m(m-3) - 2(m-3) = 0$$

$$\Rightarrow m = 3 \quad m = 2.$$

For y_p , we make the eq (1) standard.

$$\text{i.e. } y'' - \frac{4}{x}y' + \frac{6}{x^2}y = 21x^{-6}$$

Now

$$y_p = -x^2 \int \frac{x^3 \cdot 21x^{-6}}{W} dx + x^3 \int \frac{x^2 \cdot 21x^{-6}}{W} dx$$

Now

$$W = \begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix} = 3x^4 - 2x^4 = x^4$$

$$\therefore y_p = -x^2 \int \frac{x^3 \cdot 21x^{-6}}{x^4} dx + x^3 \int \frac{x^2 \cdot 21x^{-6}}{x^4} dx$$

$$\Rightarrow y_p = -x^2 \int x^{-7} dx + 21x^3 \int x^{-8} dx$$

$$\Rightarrow y_p = -21x^2 \cdot \frac{x^{-6}}{-6} + 21x^3 \cdot \frac{x^{-7}}{-7}$$

$$\Rightarrow y_p = \frac{21}{6} x^{-4} - \frac{21}{7} x^{-4}$$

$$\Rightarrow y_p = \frac{21}{42} x^{-4} = \frac{1}{2} x^{-4}$$

$$\therefore y = y_h + y_p = C_1 x^2 + C_2 x^3 + \frac{1}{2} x^{-4}$$

$$\text{(12) } xy'' - xy' = (3+x)x^3 e^x. \quad \text{--- (1) } \text{Ans}$$

For y_h , we have

$$m^2 + (-1-1)m + 0 = 0$$

$$\Rightarrow m^2 - 2m = 0 \Rightarrow m(m-2) = 0 \Rightarrow m = 0, 2$$

$$\therefore y_h = C_1 x^0 + C_2 x^2$$

(CASE - I)

Also

$$W = \begin{vmatrix} 1 & x^2 \\ 0 & 2x \end{vmatrix} = 2x$$

$$\text{i.e. } y'' - y'/x = (3+x)xe^x.$$

$$\therefore y_p = - \int \frac{x^2(3+x)xe^x dx}{2x} + x^2 \int \frac{(3+x)xe^x dx}{2x}$$

$$\Rightarrow y_p = -\frac{1}{2} \int (3x^2 + x^3)e^x dx + \frac{x^2}{2} \int (3+x)e^x dx.$$

$$\Rightarrow y_p = -\frac{1}{2} \left[(3x^2 + x^3)e^x - \int e^x (6x + 3x^2) dx \right] + \frac{x^2}{2} \left[(3+x)e^x - \int e^x dx \right]$$

$$\Rightarrow y_p = -\frac{1}{2} \left[(3x^2 + x^3)e^x - \left[e^x (6x + 3x^2) - \int e^x (6 + 6x) dx \right] \right] + \frac{x^2}{2} \left[(3+x)e^x - e^x \right]$$

$$\Rightarrow y_p = -\frac{1}{2} (3x^2 + x^3)e^x + \frac{e^x (6x + 3x^2)}{2} - \frac{1}{2} \left[e^x (6 + 6x) - \int e^x dx \right] + \frac{x^2}{2} (3+x)e^x - \frac{x^2}{2} e^x$$

$$\Rightarrow y_p = -\frac{1}{2} (3x^2 + x^3)e^x + \frac{e^x (6x + 3x^2)}{2} - \frac{1}{2} e^x (6 + 6x) + \frac{3}{2} e^x + \frac{x^2}{2} (3+x)e^x - \frac{x^2}{2} e^x$$

$$\Rightarrow y_p = \left(-\frac{3}{2}x^2 - \frac{1}{2}x^3 + 3x + \frac{3}{2}x^2 - \frac{3}{2}x + \frac{3}{2}x^2 + \frac{x^3}{2} - \frac{x^2}{2} \right) e^x$$

$$\Rightarrow y_p = 3x^2 e^x.$$

$$\therefore y = y_h + y_p = e^x + (c_1 + 3x^2)e^x$$

$$\Rightarrow y = y_h + y_p = c_1 + c_2 x^2 + 3x^2 e^x.$$

$$(13) 4x^2 y'' + 8xy' - 3y = 7x^2 - 15x^3 \quad \text{--- (1)}$$

For y_h , we have

$$x^2 y'' + 2xy' - 3/4 y = \left(7/4 x^2 - 15/4 x^3 \right)$$

and hence

$$m^2 + (2-1)m - 3/4 = 0.$$

$$\Rightarrow m^2 + m - 3/4 = 0 \Rightarrow m = \frac{-1 \pm \sqrt{1+3}}{2}$$

$$\Rightarrow m = \frac{-1 \pm 2}{2} = \frac{1}{2}, -3/2.$$

$$\therefore y_h = c_1 x^{1/2} + c_2 x^{-3/2}.$$

For y_p , we have to make (1) standard.

$$\text{i.e. } y'' + \frac{2}{x} y' - \frac{3}{4x^2} y = \frac{7}{4} - \frac{15x}{4} \quad \text{--- (2)}$$

$$\therefore y_p = -y_1 \int \frac{y_2 \left(\frac{7-15x}{4} \right) dx}{W} + y_2 \int \frac{y_1 \left(\frac{7-15x}{4} \right) dx}{W}$$

$$\Rightarrow W = \begin{vmatrix} x^{1/2} & x^{-3/2} \\ \frac{1}{2} x^{-1/2} & -\frac{3}{2} x^{-5/2} \end{vmatrix} = -\frac{3}{2} x^{-2} - \frac{1}{2} x^{-2} = -2x^{-2}$$

$$\therefore y_p = -\frac{x^{1/2}}{4} \int \frac{x^{-3/2} (7-15x) dx}{-2x^{-2}} + \frac{x^{-3/2}}{4} \int \frac{x^{1/2} (7-15x) dx}{-2x^{-2}}$$

$$\Rightarrow y_p = +\frac{x^{1/2}}{8} \int x^{1/2} (7-15x) dx + \frac{x^{-3/2}}{8} \int x^{5/2} (7-15x) dx$$

$$\Rightarrow y_p = \frac{x^{1/2}}{8} \int (7x^{1/2} - 15x^{3/2}) dx + \frac{x^{-3/2}}{8} \int (7x^{5/2} - 15x^{7/2}) dx$$

$$\Rightarrow y_p = \frac{x^{1/2}}{8} \left[\frac{7x^{3/2}}{3/2} - \frac{15x^{5/2}}{5/2} \right] + \frac{x^{-3/2}}{8} \left[\frac{7x^{7/2}}{7/2} - \frac{15x^{9/2}}{9/2} \right]$$

$$\Rightarrow y_p = \frac{x^{1/2}}{4} \left[\frac{7}{3} x^{3/2} - \frac{3}{2} x^{5/2} \right] + \frac{x^{-3/2}}{4} \left[x^{7/2} - \frac{5}{3} x^{9/2} \right]$$

$$\Rightarrow y_p = \frac{7}{12} x^2 - \frac{3}{4} x^3 - \frac{x^2}{4} + \frac{5}{12} x^3$$

$$\Rightarrow y_p = \frac{1}{3} x^2 - \frac{1}{3} x^3$$

$$\therefore y = y_h + y_p = C_1 x^{1/2} + C_2 x^{-3/2} + \frac{1}{3} x^2 - \frac{1}{3} x^3$$

Ans

P-7-0 →

Q#14 $(x^2 D^2 - 4x D + 6)y = 7x^4 \sin x.$

for y_h , we have.

\therefore Q14 $m^2 + (-4-1)m + 6 = 0 \Rightarrow m^2 - 5m + 6 = 0.$

$\Rightarrow m^2 - 3m - 2m + 6 = 0 \Rightarrow m(m-3) - 2(m-3) = 0.$

$\Rightarrow m = 2, 3.$

hence $y_h = C_1 x^2 + C_2 x^3.$

Also

$W = \begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix} = 3x^4 - 2x^4 = x^4.$

Hence $y_p = -x^2 \int \frac{x^2 \cdot 7x^4 \sin x}{x^4} dx + x^3 \int \frac{x^3 \cdot 7x^4 \sin x}{x^4} dx$

$\Rightarrow y_p = -x^2 \int 7x \sin x dx + x^3 \int 7 \sin x dx.$

$\Rightarrow y_p = -x^2 [-7x \cos x - \int -7 \cos x dx] + x^3 (-7 \cos x)$

$\Rightarrow y_p = 7x^3 \cos x - 7x^2 \sin x - 7x^3 \cos x.$

$\Rightarrow y_p = -7x^2 \sin x.$

$\therefore y = y_h + y_p = C_1 x^2 + C_2 x^3 - 7x^2 \sin x. \text{ Ans}$

Q#15 $(x^2 D^2 - 2x D + 2)y = x^3 \cos x$

for y_h , we have.

$m^2 - 3m + 2 = 0 \Rightarrow m^2 - 2m - m + 2 = 0 \Rightarrow m(m-2) - 1(m-2) = 0$

$\Rightarrow m = 1, 2. \text{ (CASE I)}$

$\therefore y_h = C_1 x + C_2 x^2.$

Also

$W = \begin{vmatrix} x & x^2 \\ 1 & 2x \end{vmatrix} = 2x^2 - x^2 = x^2.$

hence.

$y_p = -x \int \frac{x^2 \cdot x^3 \cos x}{x^2} dx + x^2 \int \frac{x \cdot x^3 \cos x}{x^2} dx.$

$\Rightarrow y_p = -x \int x \cos x dx + x^2 \int \cos x dx.$

$\Rightarrow y_p = -x [x \sin x - \int \sin x dx] + x^2 \sin x.$

$\Rightarrow y_p = -x^2 \sin x + x(-\cos x) + x^2 \sin x$

$\Rightarrow y_p = -x \cos x.$

Ans

16) $(x^2 D^2 + xD - 1)y = \frac{1}{x^2}$

Sol for y_h , we have,

$$m^2 + (1-1)m - 1 = 0 \Rightarrow m^2 - 1 = 0 \Rightarrow m = \pm 1.$$

Q16

$$\therefore y_h = c_1 x + c_2 x^{-1}.$$

Also $W = \begin{vmatrix} x & x^{-1} \\ 1 & -\frac{1}{x^2} \end{vmatrix} = -\frac{1}{x} - \frac{1}{x} = -\frac{2}{x}$

$$\therefore y_p = -x \int \frac{\frac{1}{x} \cdot \frac{1}{x^2}}{-\frac{2}{x}} dx + \frac{1}{x} \int \frac{x \cdot \frac{1}{x^2}}{-\frac{2}{x}} dx.$$

$$\Rightarrow y_p = +\frac{x}{2} \int \frac{1}{x^4} dx - \frac{1}{2x} \int \frac{1}{x^2} dx.$$

$$\Rightarrow y_p = \frac{x}{2} \frac{x^{-3}}{-3} - \frac{1}{2x} \frac{x^{-1}}{-1}$$

$$\Rightarrow y_p = \frac{-1}{6x^2} + \frac{1}{2x^2} = \frac{1}{3x^2}$$

$$\therefore y = y_h + y_p = c_1 x + \frac{c_2}{x} + \frac{1}{3x^2} \quad \underline{\text{Ans}}$$

17) $(x^2 D^2 + xD - 9)y = 48x^5$

for y_h , we have.

$$m^2 + (1-1)m - 9 = 0 \Rightarrow m = \pm 3$$

$$\therefore y_h = c_1 x^3 + c_2 x^{-3}.$$

Also

$$W = \begin{vmatrix} x^3 & x^{-3} \\ 3x^2 & -3x^{-4} \end{vmatrix} = -3x^{-1} - 3x^{-1} = -\frac{6}{x}$$

$$\therefore y_p = -x^3 \int \frac{x^{-3} \cdot 48x^5}{-\frac{6}{x}} dx + x^{-3} \int \frac{x^3 \cdot 48x^5}{-\frac{6}{x}} dx.$$

$$\Rightarrow y_p = \frac{48x^3}{6} \int x dx - \frac{48}{6} \int x^7 dx$$

$$\Rightarrow y_p = 8x^3 \left[\frac{x^2}{2} \right] - 8x^3 \left[\frac{x^8}{8} \right].$$

$$\Rightarrow y_p = 4x^5 - x^5 = 3x^5.$$